

# A MODIFIED MATRIX PENCIL MOMENT METHOD FOR MULTIMODE WAVEGUIDE DISCONTINUITIES ANALYSIS.

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## ABSTRACT

This publication presents an original approach to analyze multimode waveguide discontinuities. The generalized scattering parameters are determined by a Matrix Pencil Moment Method associated with efficient Numerically Multimode Matched Loads placed at each physical port of the discontinuities. The analysis of both microstrip-coupled lines and coplanar lines asymmetric discontinuities is presented and successfully compared to experiments and available published results.

## INTRODUCTION

With the increasing complexity of microwave and millimeter-wave integrated circuits, passive component modeling becomes more and more important in accurately determining the performance of the designed circuits, especially junction discontinuities. The integration levels of the ultra-compact MMIC's require the characterization and the modeling of multiple line interconnections. Consequently, a multiple mode problem has to be considered for many topologies. As example, a coupled microstrip line supports two quasi-TEM modes [1]. Then, a parasitic mode coupling can occur when an asymmetric discontinuity appears in the circuit. Today, planar MMIC's are more and more realized by using the coplanar waveguide [2]. The coplanar mode and the slot-line mode are the fundamental modes of the CPW. The parallel-plate mode (or substrate mode) only occurs in conductor backed coplanar transmission lines. In conductor backed coplanar MMIC's, power from the coplanar mode can be transferred to the parallel-plate mode by the leakage effect and coupling between the modes at transmission line discontinuities [3]. The excitation of the slotline mode is also a main problem if asymmetric discontinuities, like bends, are built [4] [5]. Air bridges are commonly used to suppress the slotline mode. However, the achievement of

topologies used in order to suppress the mode coupling can be appreciated only if we consider that multiple modes are propagated by the waveguide.

Generally, open and short-circuited terminations are largely involved in full-wave electromagnetic analysis. The scattering parameters of devices are then deduced from the calculated densities of surface currents flowing on the structure. They generally require a nondirect calculation involving de-embedding procedures in order to compensate for the effect of opens or shorts. Several simulations are needed to compute the whole scattering parameter matrix. Moreover, with the operating frequency increase, high reflection coefficients are expected to spoil the accuracy of the calculations.

This communication presents a full-wave analysis of multiple mode waveguide discontinuities using a Spectral Domain Approach. Based on an Electric (or Magnetic) Field Integral Equation formulation, the electric (or magnetic) current distribution on the device is solved by the well-known Galerkin's Moment Method [6]. The generalized scattering parameters are numerically measured with the help of efficient Numerically Multimode Matched Loads (NMML) placed at each physical port of the discontinuities. With that in mind, the Matrix Pencil technique is used to decompose the currents along the terminating lines into forward travelling waves and the currents along the feeding lines into forward and backward travelling waves, yielding scattering parameters of device without de-embedding procedure [7]. Numerical results and validating examples are presented. In this case, both microstrip coupled lines and coplanar lines asymmetric discontinuities are simulated. A Multimode T.R.L. calibration is also used to derive experimental multimode S parameters [8]. Our measurements and Full-Wave results are in excellent agreement. The results are in agreement also with published data [5].

## THEORY

### A Electric Field Integral Equation Formulation:

Since the Method of Moments is well known, it is only discussed briefly in order to set the notation and to note some salient points.

Figure 1 shows the basic studied multiple mode device. A microstrip line is connected to a coupled microstrip line. The asymmetry of this device implies that the two fundamental modes will be propagated in the outcoming waveguide. We have to notice that the ground plane and the dielectric substrate extend to infinity in the x and y directions. The conductors are assumed to be perfect and of zero thickness.

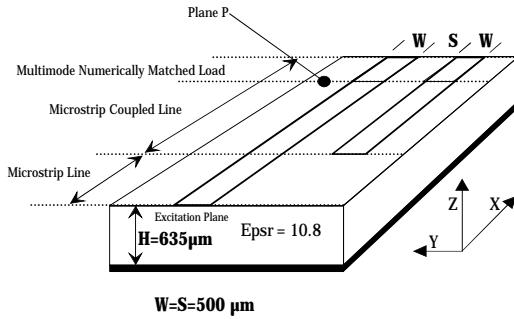


Figure 1 : A multimode structure.

The electromagnetic field problem associated with the structure shown in Figure 1 is described by an Electric Field Integral Equation (EFIE) for the surface current distribution. The previous integral equation is expressed in the Fourier domain and solved using the method of moments where electric current densities are expanded in terms of rooftop basis functions. Then, Galerkin's method is applied to project the integral equation onto a system of linear equations in the form:

$$\begin{bmatrix} Z_{ij} & \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix} = \begin{bmatrix} V \end{bmatrix}, \quad (1)$$

Weighting coefficients  $I_1, \dots, I_N$  constitute the unknowns of the problem. The excitation is modeled by ideal x-directed voltage sources located at specific node points.

### B Numerically Multimode Matched Load formulation:

In order to simplify the formulation presentation, a simpler division using only one cell along the strip width is employed. Because rooftop

functions are overlapped over two cells, only longitudinal component  $J_x$  of the current density is taken into account as shown in Figure 2.

Assuming that only the two modes propagate on the lossless microstrip coupled line, the longitudinal current distributions can be seen as the sum of both two incidents and reflected currents. As shown in figures 1 and 2, P is the reference plane where the NMML connects.

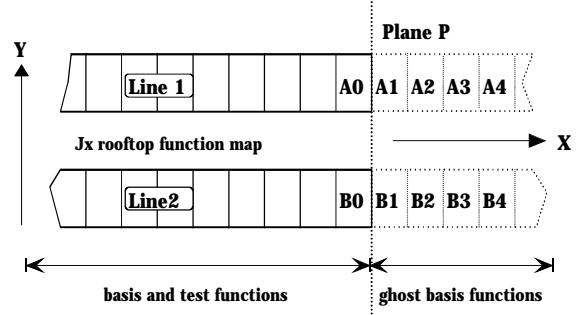


Figure 2: Current density rooftop function map.

The last  $J_x$  basis functions before the matched load are denoted  $A_0$  in the line 1,  $B_0$  in the line 2, associated with the weighting coefficients  $I_{A0}$  and  $I_{B0}$ . The matched load is modeled by forcing the current density, behind reference plane P, to belong to a traveling wave. With that in mind, rooftop  $J_x$  basis functions are added to the lines and have unknown weighting coefficients  $I_{Ai}$ ,  $I_{Bi}$  ( $i=1,2,3,\dots$ ). The longitudinal current density amplitude flowing over an infinite line has the following expression:

$$\tilde{J}_x(x) = \vec{a}_e \cdot e^{-2\pi j \frac{x}{\lambda g_e}} + \vec{a}_o \cdot e^{-2\pi j \frac{x}{\lambda g_o}} \quad (2)$$

Where  $a_e$  and  $a_o$  are related to the even and odd modes.

For monomode circuits, the unknown weighting coefficients of added basis functions might be related to each other with a simple recurrent relation. Several current densities with a different amplitude and guided wavelength are superimposed. So, the NMML must to be built with more complicated coupled recurrent relations. In the particular case of a coupled microstrip line, two coupled recurrent relations are need. This formulation is now presented.

$I_{A0}$  can be expressed as:

$$I_{A0} = \left( \frac{I_{B0} + I_{A0}}{2} \right) - \left( \frac{I_{B0} - I_{A0}}{2} \right) \quad (3)$$

$I_{B0}$  can be expressed as:

$$I_{B0} = \left( \frac{I_{B0} + I_{A0}}{2} \right) + \left( \frac{I_{B0} - I_{A0}}{2} \right) \quad (4)$$

According to the symmetry properties  $(I_{B0} + I_{A0})/2$  and  $(I_{B0} - I_{A0})/2$  are respectively the even and odd mode current density amplitudes. According to the relation 2, weighting coefficients of added basis functions  $A_i, B_i$ , may be related to each other with two coupled recurrent relations as:

$$\begin{aligned} I_{Bn} &= \frac{I_{B0}}{2} \left( e^{-2\pi j \frac{Wx}{\lambda_{ge}}} + e^{-2\pi j \frac{Wx}{\lambda_{go}}} \right) + \frac{I_{A0}}{2} \left( e^{-2\pi j \frac{Wx}{\lambda_{ge}}} - e^{-2\pi j \frac{Wx}{\lambda_{go}}} \right) \\ I_{An} &= \frac{I_{B0}}{2} \left( e^{-2\pi j \frac{Wx}{\lambda_{ge}}} - e^{-2\pi j \frac{Wx}{\lambda_{go}}} \right) + \frac{I_{A0}}{2} \left( e^{-2\pi j \frac{Wx}{\lambda_{ge}}} + e^{-2\pi j \frac{Wx}{\lambda_{go}}} \right) \end{aligned} \quad (5)$$

where  $Wx$  is the length of a unit rectangular cell with respect to the  $x$  direction.

Under this condition, the weighting coefficients of the added basis functions are related to the  $I_{A0}, I_{B0}$  unknown coefficients, so test functions over the matched load are useless. We have to notice that with the previous relations (5), the final matrix system size does not inflate with the presence of Multimode Numerically Matched Loads. The final matrix system of the problem is, with regard to the one without matched loads, changed by the accounting of the electric field diffracted by ghost basis functions on a numerically matched load. For each test function  $i$  lying on the studied structure, the reaction terms related to the unknown weighting coefficients of the last basis functions,  $I_{A0}$  and  $I_{B0}$ , are changed in agreement with relations (5), and the matrix system to solve becomes:

$$\begin{bmatrix} 1 & A0 & B0 \\ Z_{11} & \vdots & \vdots \\ \vdots & \vdots & \vdots \\ i & \dots & Z_{i,A0} & \dots & Z_{i,B0} & \dots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix} \begin{bmatrix} I_1 \\ \vdots \\ I_{A0} \\ \vdots \\ I_{B0} \\ \vdots \end{bmatrix} = \begin{bmatrix} V_1 \\ \vdots \\ V_2 \\ \vdots \end{bmatrix} \quad (6)$$

As example:

$$\begin{aligned} Z_{i,A0} &= Z_{i,A0} + \frac{1}{2} \left[ Z_{i,A1} \left( e^{-2\pi j \frac{Wx}{\lambda_{ge}}} + e^{-2\pi j \frac{Wx}{\lambda_{go}}} \right) + Z_{i,B1} \left( e^{-2\pi j \frac{Wx}{\lambda_{ge}}} - e^{-2\pi j \frac{Wx}{\lambda_{go}}} \right) \right] \\ &+ \frac{1}{2} \left[ Z_{i,A2} \left( e^{-4\pi j \frac{Wx}{\lambda_{ge}}} + e^{-4\pi j \frac{Wx}{\lambda_{go}}} \right) + Z_{i,B2} \left( e^{-4\pi j \frac{Wx}{\lambda_{ge}}} - e^{-4\pi j \frac{Wx}{\lambda_{go}}} \right) \right] + \dots \end{aligned}$$

When a more detailed division will be necessary to include the longitudinal current component, this numerical procedure can be easily extended. The NMML can be also extended to coplanar lines.

## NUMERICAL RESULTS

We propose in a first example to demonstrate the efficiency of the NMML. In this mind, we show in figure 3 the current density magnitudes of the two modes supported by the structure shown in figure 1. These current densities are extracted from the total current density with the Matrix Pencil. As shown in figure 3, a traveling-wave configuration is obtained for the two electric surface current distributions before the NMML and validates our numerical simulation.

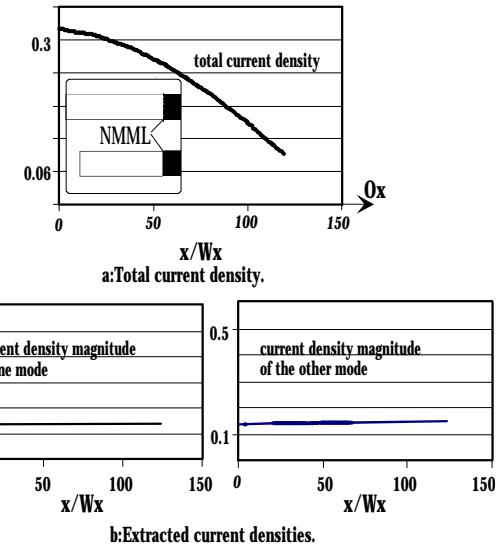


Figure 3 : Magnitudes of the current densities.

The second validation example is an asymmetric microstrip coupled line open-end shown in figure 4. In this case, mode conversion leads to a  $2 \times 2$  scattering matrix between the modal amplitudes:

$$\begin{bmatrix} b_e \\ b_o \end{bmatrix} = \begin{bmatrix} R_{ee} & R_{eo} \\ R_{oe} & R_{oo} \end{bmatrix} \begin{bmatrix} a_e \\ a_o \end{bmatrix} \quad (7)$$

For example,  $R_{ee}$  is the reflection coefficient of the odd quasi-TEM mode due to the incident even quasi-TEM mode of unit amplitude. The NMML is placed before the delta gap voltage generators in order to avoid reflection waves from the excitation mechanism. For the sake of comparison, an analytical model is also applied to analyze the mode conversion. A multimode TRL calibration is also used to derive experimental multimode S parameters [8]. These

measurements are in excellent agreement with analytical and full-wave results.

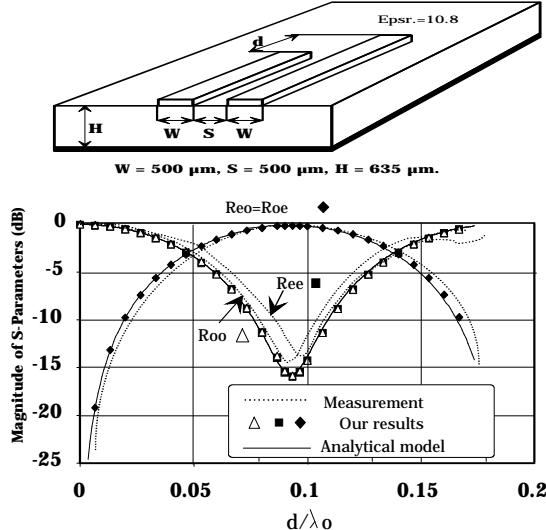


Figure 4: Magnitude of the reflection coefficients.

In a third example, a CPW bend, shown in figure 4, is also studied and compared with success to available results [5].

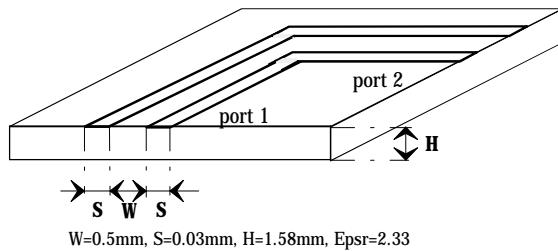
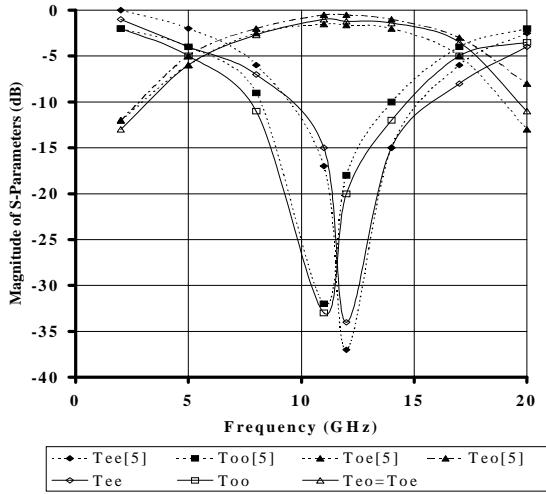


Figure 5: Magnitude of the transmission coefficients for the CPW bend.



The figure 5 represents the magnitude of the transmission coefficients  $T_{i,j}$  ( $i,j = o,e$ ) versus frequency. The subscript  $o$  is associated with the odd slotline mode of the CPW. The subscript  $e$  is associated with the even coplanar mode of the CPW.

## CONCLUSION

We propose in this communication to present a multimode waveguide discontinuities analysis. A powerful technique is obtained by combining a Spectral Domain Moment Method, the Matrix Pencil technique and an original Multimode Numerically Matched Load formulation. Comparisons of numerical results with experimental and available results show good agreement.

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